

## Analysis of Regression Discontinuity Designs with Multiple Cutoffs or Multiple Scores

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**Abstract.** We introduce the **Stata** (and **R**) package **rdmulti**, which includes three commands (**rdmc**, **rdmcpplot**, **rdms**) for analyzing Regression Discontinuity (RD) designs with multiple cutoffs or multiple scores. The command **rdmc** applies to non-cumulative and cumulative multi-cutoff RD settings. It calculates pooled and cutoff-specific RD treatment effects, and provides robust bias-corrected inference procedures. Post estimation and inference is allowed. The command **rdmcpplot** offers RD plots for multi-cutoff settings. Finally, the command **rdms** concerns multi-score settings, covering in particular cumulative cutoffs and two running variables contexts. It also calculates pooled and cutoff-specific RD treatment effects, provides robust bias-corrected inference procedures, and allows for post-estimation estimation and inference. These commands employ the **Stata** (and **R**) package **rdrobust** for plotting, estimation, and inference. Companion **R** functions with the same syntax and capabilities are provided.

**Keywords:** st0001, regression discontinuity designs, multiple cutoffs, multiple scores, local polynomial methods.

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# 1 Introduction

Regression discontinuity (RD) designs with multiple cutoffs or multiple scores are commonly encountered in empirical work in Economics, Education, Political Science, Public Policy, and many other disciplines. As a consequence, these specific settings have also received attention in the recent RD methodological literature (Papay, Willett, and Murnane 2011; Reardon and Robinson 2012; Wong, Steiner, and Cook 2013; Keele and Titiunik 2015; Keele, Titiunik, and Zubizarreta 2015; Cattaneo, Keele, Titiunik, and Vazquez-Bare 2016a, 2021, and references therein). In this article, we introduce the software package `rdmulti`, which includes three `Stata` commands (and analogous R functions) for the analysis of RD designs with multiple cutoffs or multiple scores.

The command `rdmc` applies to non-cumulative and cumulative multi-cutoff RD settings, following recent work in Cattaneo, Keele, Titiunik, and Vazquez-Bare (2016a, 2021). Specifically, it calculates pooled and cutoff-specific RD treatment effects, employing local polynomial estimation and robust bias-corrected inference procedures. Post estimation and inference is allowed. The companion command `rdmcpplot` offers RD plots for multi-cutoff settings. Finally, the command `rdms` concerns multi-score settings, covering in particular cumulative cutoffs and bivariate score contexts. It also calculates pooled and cutoff-specific RD treatment effects based on local polynomial methods, and allows for post-estimation estimation and inference. These commands employ the `Stata` (and R) package `rdrobust` for plotting, estimation, and inference; see Calonico, Cattaneo, and Titiunik (2014a, 2015b); Calonico, Cattaneo, Farrell, and Titiunik (2017) for software details. See also Cattaneo, Titiunik, and Vazquez-Bare (2017) for a comparison of RD methodologies, Cattaneo, Idrobo, and Titiunik (2019a, 2020) and Cattaneo, Titiunik, and Vazquez-Bare (2019b) for a practical introductions to RD designs, and Cattaneo and Escanciano (2017) for a recent edited volume with further references.

To streamline the presentation, this article employs only simulated data to showcase all three settings covered by the package `rdmulti`: non-cumulative multiple cutoffs, cumulative multiple cutoffs, and bivariate score settings. For further discussion and illustration employing real data sets see Cattaneo, Idrobo, and Titiunik (2020). The three settings covered by the package correspond, respectively, to (i) RD designs where different subgroups in the data are exposed to distinct but only one of the cutoff points (non-cumulative case), (ii) RD designs where units receive one single score and units are confronted to a sequence of ordered cutoffs points (cumulative case), and (iii) RD designs where units received two scores and there is a boundary on the plane determining the control and treatment areas. Well-known examples of each of these settings are:

- *Non-Cumulative Multiple Cutoffs*: units in different groups (e.g., schools) receive an univariate score (e.g., test score) but the RD cutoff varies by group;
- *Cumulative Multiple Cutoffs*: units receive the an univariate score (e.g., age) but different treatments are assigned at distinct score levels (e.g., at age 60 and at age 65);
- *Multiple Scores*: units receive two scores (e.g., latitude and longitude) and treatment is assigned based on a boundary depending on both scores (e.g., geographic boundary).

We elaborate further on these cases in the upcoming sections, where we also give graphical representations of each case.

The **Stata** (and **R**) package **rdmulti** complements several recent software packages for RD designs. First, it explicitly relies on **rdrobust** (Calonico, Cattaneo, and Titiunik 2014a, 2015b; Calonico, Cattaneo, Farrell, and Titiunik 2017) for implementation, and hence further extends its scope to the case of RD designs with multiple cutoffs or multiple scores. Second, while the package focuses on local polynomial methods, related methods employing local randomization ideas and implemented in the package **rdlocrand** can also be used in the contexts of multiple cutoffs and multiple scores (Cattaneo, Titiunik, and Vazquez-Bare 2016b). Third, the package **rddensity** (Cattaneo, Jansson, and Ma 2018) can also be used in multiple cutoffs or multiple scores settings for falsification purposes. Finally, see the package **rdpower** (Cattaneo, Titiunik, and Vazquez-Bare 2019c) for power calculations and sampling design methods, which can also be applied in the contexts discussed in this article.

The rest of the article is organized as follows. Section 2 gives a brief overview of the methods implemented in the package **rdmulti**, and also provides further references. Sections 3, 4 and 5 discuss the syntax of the commands **rdmc**, **rdmcplot** and **rdms**, respectively. Section 6 gives numerical illustrations, and Section 7 concludes. The latest version of this software, as well other software and materials useful for the analysis of RD designs, can be found at:

<https://sites.google.com/site/rdpackages/>

## 2 Overview of Methods

In this section we briefly describe the main ideas and methods used in the package **rdmulti**. For further methodological details see Keele and Titiunik (2015), Cattaneo, Keele, Titiunik, and Vazquez-Bare (2016a, 2021), Cattaneo, Idrobo, and Titiunik (2020), and references therein. All estimation and inference procedures employ **rdplots** (Calonico, Cattaneo, and Titiunik 2015a) as well as local polynomial point estimation and robust bias correction inference methods (Calonico, Cattaneo, and Titiunik 2014b; Calonico, Cattaneo, and Farrell 2018; Calonico, Cattaneo, Farrell, and Titiunik 2019; Calonico, Cattaneo, and Farrell 2020b,a).

### 2.1 Non-cumulative Multiple Cutoffs

In this case, individuals have a running variable  $X_i$  and a vector of potential outcomes  $(Y_i(0), Y_i(1))$ . Each individual faces a cutoff  $C_i \in \mathcal{C}$  with  $\mathcal{C} = \{c_1, c_2, \dots, c_J\}$ . For example, Chay et al. (2005) study the effect of a school improvement program introduced in 1990 by the Chilean government. In this program, low-performing schools received public funding to improve infrastructure and teacher training, among other things. Assignment to this program was based on a school-level measure of test scores falling below a cutoff, where the cutoff was different across Chile's 13 administrative regions. In this example,  $C_i$  indicates each school's administrative region, since this determines

the cutoff faced by each school.

Unlike in a standard single-cutoff RD design,  $C_i$  is a random variable. In a sharp design, individuals are treated when their running variable exceeds their corresponding cutoff,  $D_i = \mathbb{1}(X_i \geq C_i)$ . A key feature of this design is that the variable  $C_i$  partitions the population, that is, each unit faces one and only one value of  $C_i$ . As the notation suggests, the potential outcomes for each individual are the same regardless of the specific cutoff they are exposed to; see [Cattaneo, Keele, Titiunik, and Vazquez-Bare \(2016a, 2021\)](#) for more discussion. Finally, we only consider finite multiple cutoffs because this is the most natural setting for empirical work: in practice, continuous cutoff are discretized for estimation and inference, as discussed and illustrated below.

Under regularity conditions, which include smoothness of conditional expectations among other things (see aforementioned references for details), the cutoff-specific treatment effects,  $\tau(c) = \mathbb{E}[Y_i(1) - Y_i(0) | X_i = c, C_i = c]$ , are identified by:

$$\tau(c) = \lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x, C_i = c] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x, C_i = c] \quad (1)$$

The pooled RD estimate is obtained by recentering the running variable,  $\tilde{X}_i = X_i - C_i$ , thus normalizing the cutoff at zero:

$$\tau_P = \lim_{x \downarrow 0} \mathbb{E}[Y_i | \tilde{X}_i = x] - \lim_{x \uparrow 0} \mathbb{E}[Y_i | \tilde{X}_i = x], \quad (2)$$

where

$$\tau_P = \sum_{c \in \mathcal{C}} \tau(c) \omega(c), \quad \omega(c) = \frac{f_{X|C}(c|c) \mathbb{P}[C_i = c]}{\sum_{c \in \mathcal{C}} f_{X|C}(c|c) \mathbb{P}[C_i = c]} \quad (3)$$

All these parameters can be readily estimated using local polynomial methods (see [Cattaneo, Idrobo, and Titiunik 2019a](#), for a practical introduction), conditioning on cutoffs when appropriate. In other words, RD methods can be applied to each cutoff separately, in addition to pooling the data. Therefore, the `rdmulti` package implements bandwidth selection, estimation and inference based on local polynomial methods using the `rdrobust` command, described in [Calonico et al. \(2014a, 2015b, 2017\)](#). Specifically, the command `rdmc` allows for multi-cutoffs RD designs.

For the pooled parameter  $\tau_P$ , the weights are estimated using the fact that  $\omega(c) = \mathbb{P}[C_i = c | \tilde{X}_i = 0]$ ; see [Cattaneo, Keele, Titiunik, and Vazquez-Bare \(2016a\)](#) for further details. Then, given a bandwidth  $h > 0$ ,

$$\hat{\omega}(c) = \frac{\sum_i \mathbb{1}(C_i = c, -h \leq \tilde{X}_i \leq h)}{\sum_i \mathbb{1}(-h \leq \tilde{X}_i \leq h)}.$$

When not specified by the user, the `rdmc` command uses the bandwidth selected by `rdrobust` when estimating the pooled effect to estimate the weights.

## 2.2 Cumulative Multiple Cutoffs

In an RD setting with cumulative cutoffs, individuals receive different treatments (or different dosages of a treatment) for different ranges of the running variable. In such setting, individuals receive treatment 1 if  $X_i < c_1$ , treatment 2 if  $c_1 \leq X_i < c_2$ , and so on, until the last treatment value at  $X_i \geq c_J$ . For example, [Brollo et al. \(2013\)](#) examine the effect of federal transfers on political corruption in Brazilian municipalities. The amount of the federal transfer that municipalities receive depends on the municipality's population, and changes discretely at specified cutoffs. For example, municipalities with population below 10,189 receive a certain amount, municipalities with population between 10,189 and 13,585 receive a larger amount, and so on.

Denote the values of these treatments as  $d_j$ , so that the treatment variable is now  $D_i \in \{d_1, d_2, \dots, d_J\}$ . Under standard regularity conditions, we have:

$$\tau_j = \mathbb{E}[Y_i(d_j) - Y_i(d_{j-1}) | X_i = c_j] = \lim_{x \downarrow c_j} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c_j} \mathbb{E}[Y_i | X_i = x]$$

Since, unlike the case with multiple non-cumulative cutoffs, the population is not partitioned, each observation can be used to estimate two different (but contiguous on the score dimension) treatment effects. For example, units receiving treatment dosage  $d_j$  are used as “treated” (i.e. above the cutoff  $c_j$ ) when estimating  $\tau_j$  and as “controls” when estimating  $\tau_{j+1}$  (i.e. below the cutoff  $c_{j+1}$ ). As a result, cutoff-specific estimators may not be independent, although the dependence disappears asymptotically as long as the bandwidths around each cutoff decrease with the sample size. On the other hand, bandwidths can be chosen to be non-overlapping to ensure that observations are used only once.

Once the data has been assigned to each cutoff under analysis, local polynomial methods can also be applied cutoff by cutoff in the cumulative multiple cutoffs case. We illustrate this approach below; for further discussion see [Cattaneo, Idrobo, and Titiunik \(2020\)](#), and the references therein.

## 2.3 Multiple Scores

In a multi-score RD design, treatment is assigned based on multiple running variables and some function determining a treatment “region” or “area”. We focus on the case with two running variables,  $\mathbf{X}_i = (X_{1i}, X_{2i})$ , which is by far the most common case in empirical work. This case occurs naturally when, for instance, a treatment is assigned based on scores in two different exams (such as language and mathematics). [Matsudaira \(2008\)](#) estimates the effect of a mandatory summer school program assigned to students who fail to score higher than a preset cutoff in both math and reading exams. Another common case of multiple running variables occurs when a treatment is assigned based on geographic location (e.g., latitude and longitude). [Keele and Titiunik \(2015\)](#) discuss the effect of political campaign advertising on voter turnout and political attitudes by comparing voters in adjacent media markets, which result in different levels of exposure to advertising.

This type of assignment defines a continuum of treatment effects over the boundary of the treatment region, denoted by  $\mathcal{B}$ . For instance, if treatment is assigned to students scoring below 50 in language and mathematics, the treatment boundary is  $\mathcal{B} = \{x_1 \leq 50, x_2 = 50\} \cup \{x_1 = 50, x_2 \leq 50\}$ . For each point  $\mathbf{b} \in \mathcal{B}$ , the treatment effect at that point is given by

$$\tau(\mathbf{b}) = \mathbb{E}[Y_i(1) - Y_i(0) | \mathbf{X}_i = \mathbf{b}],$$

and under regularity conditions,

$$\tau(\mathbf{b}) = \lim_{\substack{d(\mathbf{x}, \mathbf{b}) \rightarrow 0, \\ \mathbf{x} \in \mathcal{B}_t}} \mathbb{E}[Y_i | \mathbf{X}_i = \mathbf{x}] - \lim_{\substack{d(\mathbf{x}, \mathbf{b}) \rightarrow 0, \\ \mathbf{x} \in \mathcal{B}_c}} \mathbb{E}[Y_i | \mathbf{X}_i = \mathbf{x}]$$

where  $\mathcal{B}_c$  and  $\mathcal{B}_t$  denote the control and treatment areas, respectively, and  $d(\cdot, \cdot)$  is a metric.

Since estimating a whole curve of treatment effects may not be feasible in practice, it is common to define a set of boundary points of interest at which to estimate the RD treatment effects. In the previous example, for instance, three points of interest on the boundary determining treatment assignment could be  $\{(25, 50), (50, 50), (50, 25)\}$ . On the other hand, the pooled RD estimand requires defining some measure of distance to the cutoff, such as the perpendicular (Euclidean) distance. This distance can be seen as the recentered running variable  $\tilde{X}_i$ , which allows defining the pooled estimand as in Equation 2.

### 3 rdmc syntax

This section describes the syntax of the command `rdmc`, which estimates the pooled and cutoff-specific RD effects using `rdrobust`.

#### 3.1 Syntax

```
rdmc depvar runvar [if][in], cvar(cutoff_var) [fuzzy(string)
  derivvar(string) pooled_opt(string) verbose pvar(string)
  qvar(string) hvar(string) hrightvar(string) bvar(string)
  brightvar(string) rhovar(string) covsvar(string) covsdropvar(string)
  kernelvar(string) weightsvar(string) bwselectvar(string)
  scaleparvar(string) scaleregulvar(string) masspointsvar(string)
  bwcheckvar(string) bwrestrictvar(string) stdvarsvar(string)
  vcevar(string) level(#) plot graph_opt(string) ]
```

*depvar* is the dependent variable.

*runvar* is the running variable (a.k.a. score or forcing variable).

`cvar(cutoff_var)` specifies the numeric variable *cutoff\_var* that indicates the cutoff faced by each unit in the sample.

`fuzzy(string)` indicates a fuzzy design. See `help rdrobust` for details.

`derivvar(string)` a variable of length equal to the number of different cutoffs that specifies the order of the derivative for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

`pooled_opt(string)` specifies the options to be passed to `rdrobust` to calculate pooled estimate. See `help rdrobust` for details.

`verbose` displays the output from `rdrobust` to calculate pooled estimand.

`pvar(string)` a variable of length equal to the number of different cutoffs that specifies the order of the polynomials for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

`qvar(string)` a variable of length equal to the number of different cutoffs that specifies the order of the polynomials for bias estimation for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

`hvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths for `rdrobust` to calculate cutoff-specific estimates. When `hrightvar` is specified, `hvar` indicates the bandwidth to the left of the cutoff. When `hrightvar` is not specified, the same bandwidths are used at each side. See `help rdrobust` for details.

`hrightvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths to the right of the cutoff for `rdrobust` to calculate cutoff-specific estimates. When `hrightvar` is not specified, the same bandwidths in `hvar` are used at each side. See `help rdrobust` for details.

`bvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths for the bias for `rdrobust` to calculate cutoff-specific estimates. When `brightvar` is specified, `bvar` indicates the bandwidth to the left of the cutoff. When `brightvar` is not specified, the same bandwidths are used at each side. See `help rdrobust` for details.

`brightvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths to the right of the cutoff for `rdrobust` to calculate cutoff-specific estimates. When `brightvar` is not specified, the same bandwidths in `bvar` are used at each side. See `help rdrobust` for details.

`rhoval(string)` a variable of length equal to the number of different cutoffs that specifies the value of rho for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

`covsvar(string)` a variable of length equal to the number of different cutoffs that specifies the covariates for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

covsdropvar(*string*) a variable of length equal to the number of different cutoffs that specifies whether collinear covariates should be dropped. See **help rdrobust** for details.

kernelvar(*string*) a variable of length equal to the number of different cutoffs that specifies the kernels for **rdrubust** to calculate cutoff-specific estimates. See **help rdrobust** for details.

weightsvar(*string*) a variable of length equal to the number of different cutoffs that specifies the weights for **rdrubust** to calculate cutoff-specific estimates. See **help rdrobust** for details.

bwselectvar(*string*) a variable of length equal to the number of different cutoffs that specifies the bandwidth selection method for **rdrubust** to calculate cutoff-specific estimates. See **help rdrobust** for details.

scaleparvar(*string*) a variable of length equal to the number of different cutoffs that specifies the value of **scalepar** for **rdrubust** to calculate cutoff-specific estimates. See **help rdrobust** for details.

scaleregulvar(*string*) a variable of length equal to the number of different cutoffs that specifies the value of **scaleregul** for **rdrubust** to calculate cutoff-specific estimates. See **help rdrobust** for details.

masspointsvar(*string*) a variable of length equal to the number of different cutoffs that specifies how to handle repeated values in the running variable. See **help rdrobust** for details.

bwcheckvar(*string*) a variable of length equal to the number of different cutoffs that specifies the value of **bwcheck**. See **help rdrobust** for details.

bwrestrictvar(*string*) variable of length equal to the number of different cutoffs that specifies whether computed bandwidths are restricted to the range of *runvar*. See **help rdrobust** for details.

stdvarsvar(*string*) a variable of length equal to the number of different cutoffs that specifies whether *depvar* and *runvar* are standardized. See **help rdrobust** for details.

vcevar(*string*) a variable of length equal to the number of different cutoffs that specifies the variance-covariance matrix estimation method for **rdrubust** to calculate cutoff-specific estimates. See **help rdrobust** for details.

**level**(#) specifies the confidence levels for confidence intervals. See **help rdrobust** for details.

**plot** plots the pooled and cutoff-specific estimates and the weights given by the pooled estimate to each cutoff-specific estimate.

**graph\_opt**(*string*) options to be passed to the graph when **plot** is specified.



## 4 rdmcplot syntax

This section describes the syntax of the command `rdmcplot`, which plots the regression functions for each of the groups facing each cutoff using `rdplot`.

### 4.1 Syntax

```
rdmcplot depvar runvar [ if ] [ in ], cvar(cutoff_var) [ nbinsvar(string)
  nbinsrightvar(string) binselectvar(string)
  scalevar(string) scalerightvar(string) supportvar(string)
  supportrightvar(string) pvar(string) hvar(string) hrightvar(string)
  kernelvar(string) weightsvar(string) covsvar(string)
  covsevalvar(string) covsdropvar(string) binsoptvar(string)
  lineoptvar(string) xlineoptvar(string) ci(cilevel) nobins nopoly
  noxline nodraw genvars ]
```

*depvar* is the dependent variable.

*runvar* is the running variable (a.k.a. score or forcing variable).

`cvar(cutoff_var)` specifies the numeric variable *cutoff\_var* that indicates the cutoff faced by each unit in the sample.

`nbinsvar(string)` a variable of length equal to the number of different cutoffs that specifies the number of bins for `rdplot`. When `nbinsrightvar` is specified, `nbinsvar` indicates the number of bins to the left of the cutoff. When `nbinsrightvar` is not specified, the same number of bins is used at each side. See `help rdplot` for details.

`nbinsrightvar(string)` a variable of length equal to the number of different cutoffs that specifies the number of bins to the right of the cutoff for `rdplot`. When `nbinsrightvar` is not specified, the number of bins in `nbinsvar` used at each side. See `help rdplot` for details.

`binselectvar(string)` a variable of length equal to the number of different cutoffs that specifies the bin selection method for `rdplot`. See `help rdplot` for details.

`scalevar(string)` a variable of length equal to the number of different cutoffs that specifies the scale for `rdplot`. When `scalerightvar` is specified, `scalevar` indicates the scale to the left of the cutoff. When `scalerightvar` is not specified, the same scale is used at each side. See `help rdplot` for details.

`scalerightvar(string)` a variable of length equal to the number of different cutoffs that specifies the scale to the right of the cutoff for `rdplot`. When `scalerightvar` is not specified, the scale in `scalevar` is used at each side. See `help rdplot` for details.

`supportvar(string)` a variable of length equal to the number of different cutoffs that

specifies the support for `rdplot`. When `supportrightvar` is specified, `supportvar` indicates the support to the left of the cutoff. When `supportrightvar` is not specified, the same support is used at each side. See `help rdplot` for details.

`supportrightvar(string)` a variable of length equal to the number of different cutoffs that specifies the support to the right of the cutoff for `rdplot`. When `supportrightvar` is not specified, the support in `supportvar` is used at each side. See `help rdplot` for details.

`pvar(string)` a variable of length equal to the number of different cutoffs that specifies the order of the polynomials for `rdplot`. See `help rdplot` for details.

`hvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths for `rdplot`. When `hrightvar` is specified, `hvar` indicates the bandwidth to the left of the cutoff. When `hrightvar` is not specified, the same bandwidth is used at each side. See `help rdplot` for details.

`hrightvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidth to the right of the cutoff for `rdplot`. When `hrightvar` is not specified, the bandwidth in `hvar` is used at each side. See `help rdplot` for details.

`kernelvar(string)` a variable of length equal to the number of different cutoffs that specifies the kernels for `rdplot`. See `help rdplot` for details.

`weightsvar(string)` a variable of length equal to the number of different cutoffs that specifies the weights for `rdplot`. See `help rdplot` for details.

`covsvar(string)` a variable of length equal to the number of different cutoffs that specifies the covariates for `rdplot`. See `help rdplot` for details.

`covsevalvar(string)` a variable of length equal to the number of different cutoffs that specifies the evaluation points for additional covariates. See `help rdplot` for details.

`covsdropvar(string)` a variable of length equal to the number of different cutoffs that specifies whether collinear covariates should be dropped. See `help rdplot` for details.

`binsoptvar(string)` a variable of length equal to the number of different cutoffs that specifies options for the bins plots.

`lineoptvar(string)` a variable of length equal to the number of different cutoffs that specifies options for the polynomial plots.

`xlineoptvar(string)` a variable of length equal to the number of different cutoffs that specifies options for the vertical lines indicating the cutoffs.

`ci(cilevel)` adds confidence intervals of level `cilevel` to the plot.

`nobins` omits the bins plot.

`nopoly` omits the polynomial curve plot.

`noxline` omits the vertical lines indicating the cutoffs.

`nodraw` omits the plot.

`genvars` generates variables to replicate plots by hand. Variable labels indicate the corresponding cutoff.

`rdmcpplot_hat_y_c` predicted value of the outcome variable given by the global polynomial estimator in cutoff number  $c$ .

`rdmcpplot_mean_x_c` sample mean of the running variable within the corresponding bin for each observation in cutoff number  $c$ .

`rdmcpplot_mean_y_c` sample mean of the outcome variable within the corresponding bin for each observation in cutoff number  $c$ .

`rdmcpplot_ci_l_c` lower end value of the confidence interval for the sample mean of the outcome variable within the corresponding bin for each observation in cutoff number  $c$ .

`rdmcpplot_ci_r_c` upper end value of the confidence interval for the sample mean of the outcome variable within the corresponding bin for each observation in cutoff number  $c$ .

## 5 rdms syntax

This section describes the syntax of the command `rdms`, which analyzes RD designs with cumulative cutoffs or two running variables.

### 5.1 Syntax

```
rdms depvar runvar1 [runvar2 treatvar] [if][in], cvar(cutoff_var1
    [cutoff_var2]) [range(range1 [range2]) xnorm(string)
    fuzzy(string) derivvar(string) pooled_opt(string) pvar(string)
    qvar(string) hvar(string) hrightvar(string) bvar(string)
    brightvar(string) rho var(string) covsvar(string) covsdropvar(string)
    kernelvar(string) weightsvar(string) bwselectvar(string)
    scaleparvar(string) scaleregulvar(string) masspointsvar(string)
    bwcheckvar(string) bwrestrictvar(string) stdvarsvar(string)
    vcevar(string) level(#) plot graph_opt(string) ]
```

`depvar` is the dependent variable.

`runvar1` is the running variable (a.k.a. score or forcing variable) in a cumulative cutoffs setting.

*runvar2* if specified, is the second running variable (a.k.a. score or forcing variable) in a two-score setting.

*treatvar* if specified, is the treatment indicator in a two-score setting.

`cvar(cutoff_var1 [cutoff_var2])` specifies the numeric variable *cutoff\_var1* that indicates the cutoff faced by each unit in the sample in a cumulative cutoff setting, or the two running variables *cutoff\_var1* and *cutoff\_var2* in a two-score RD design.

`range(range1 [range2])` specifies the range of the running variable to be used for estimation around each cutoff. Specifying only one variable implies using the same range at each side of the cutoff.

`xnorm(string)` specifies the normalized running variable to estimate pooled effect.

`fuzzy(string)` indicates a fuzzy design. See `help rdrobust` for details.

`derivvar(string)` a variable of length equal to the number of different cutoffs that specifies the order of the derivative for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

`pooled_opt(string)` specifies the options to be passed to `rdrobust` to calculate pooled estimate. See `help rdrobust` for details.

`pvar(string)` a variable of length equal to the number of different cutoffs that specifies the order of the polynomials for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

`qvar(string)` a variable of length equal to the number of different cutoffs that specifies the order of the polynomials for bias estimation for `rdrobust` to calculate cutoff-specific estimates. See `help rdrobust` for details.

`hvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths for `rdrobust` to calculate cutoff-specific estimates. When `hrightvar` is specified, `hvar` indicates the bandwidth to the left of the cutoff. When `hrightvar` is not specified, the same bandwidths are used at each side. See `help rdrobust` for details.

`hrightvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths to the right of the cutoff for `rdrobust` to calculate cutoff-specific estimates. When `hrightvar` is not specified, the same bandwidths in `hvar` are used at each side. See `help rdrobust` for details.

`bvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths for the bias for `rdrobust` to calculate cutoff-specific estimates. When `brightvar` is specified, `bvar` indicates the bandwidth to the left of the cutoff. When `brightvar` is not specified, the same bandwidths are used at each side. See `help rdrobust` for details.

`brightvar(string)` a variable of length equal to the number of different cutoffs that specifies the bandwidths to the right of the cutoff for `rdrobust` to calculate cutoff-specific estimates. When `brightvar` is not specified, the same bandwidths in `bvar`

are used at each side. See `help rdrobust` for details.

`rho`var(*string*) a variable of length equal to the number of different cutoffs that specifies the value of `rho` for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`cov`svar(*string*) a variable of length equal to the number of different cutoffs that specifies the covariates for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`cov`sdropvar(*string*) a variable of length equal to the number of different cutoffs that specifies whether collinear covariates should be dropped. See `help rdrobust` for details.

`kernel`var(*string*) a variable of length equal to the number of different cutoffs that specifies the kernels for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`weights`var(*string*) a variable of length equal to the number of different cutoffs that specifies the weights for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`bw`selectvar(*string*) a variable of length equal to the number of different cutoffs that specifies the bandwidth selection method for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`scale`parvar(*string*) a variable of length equal to the number of different cutoffs that specifies the value of `scalepar` for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`scaleregul`var(*string*) a variable of length equal to the number of different cutoffs that specifies the value of `scaleregul` for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`masspoints`var(*string*) a variable of length equal to the number of different cutoffs that specifies how to handle repeated values in the running variable. See `help rdrobust` for details.

`bw`checkvar(*string*) a variable of length equal to the number of different cutoffs that specifies the value of `bwcheck`. See `help rdrobust` for details.

`bw`restrictvar(*string*) variable of length equal to the number of different cutoffs that specifies whether computed bandwidths are restricted to the range of `runvar`. See `help rdrobust` for details.

`std`varsvar(*string*) a variable of length equal to the number of different cutoffs that specifies whether `depvar` and `runvar` are standardized. See `help rdrobust` for details.

`vce`var(*string*) a variable of length equal to the number of different cutoffs that specifies the variance-covariance matrix estimation method for `rdr`obust to calculate cutoff-specific estimates. See `help rdrobust` for details.

`level(#)` specifies the confidence levels for confidence intervals. See `help rdrobust` for details.

`plot` plots the pooled and cutoff-specific estimates and the weights given by the pooled estimate to each cutoff-specific estimate.

`graph_opt(string)` options to be passed to the graph when `plot` is specified.

## 6 Illustration of Methods

### 6.1 Non-cumulative Multiple Cutoffs

We begin by illustrating `rdmc` using a simulated dataset, `simdata_multic`. In this dataset, `y` is the outcome variable, `x` is the running variable, `c` is a variable indicating the cutoff that each unit in the sample faces, and `t` is a treatment indicator, corresponding in this case to units with  $x \geq c$ . As shown below, there are two different cutoffs, 33 and 66, each with the sample size.

```
. use simdata_multic, clear
. sum
```

Variable	Obs	Mean	Std. Dev.	Min	Max
c	2,000	49.5	16.50413	33	66
x	2,000	50.79875	28.95934	.0184725	99.97507
t	2,000	.516	.4998689	0	1
y	2,000	1728.135	545.0856	540.0849	3015.232

```
. tab c
```

c	Freq.	Percent	Cum.
33	1,000	50.00	50.00
66	1,000	50.00	100.00
Total	2,000	100.00	

The basic syntax for `rdmc` is the following:

```
. rdmc y x, c(c)
```

Cutoff-specific RD estimation with robust bias-corrected inference

Cutoff	Coef.	P> z	[95% Conf. Int.]	hl	hr	Nh	Weight
33	484.831	0.00	421.18 552.53	14.66	14.66	289	0.540
66	297.981	0.00	220.35 362.27	11.95	11.95	246	0.460
Weighted	398.915	0.00	348.74 445.14	.	.	535	.
Pooled	436.400	0.00	179.34 676.63	13.68	13.68	550	.

The output shows the cutoff-specific estimate at each cutoff, together with the corresponding robust bias-corrected p-value, 95 percent robust confidence interval and sample size at each cutoff, and two “global” estimates. The first one is a weighted average of the cutoff specific estimates using the estimated weights described in Section 2. These estimated weights are shown in the last column. The second one is the pooled estimate obtained by normalizing the running variable. While these two estimators converge to

the same population parameter, they can differ in finite samples as seen above. In this example, the effect is statistically significant at both cutoffs.

All the results in the above display are calculated using `rdrobust`. The user can specify options for `rdrobust` to calculate the pooled estimates using the option `pooled_opt`. For instance, the syntax below specifies a bandwidth of 20 and a local quadratic polynomial for the pooled estimand. By default, `rdmc` omits the output from `rdrobust` when estimating the effects. The output from the pooled effect estimation can be displayed using the option `verbose`, which we use below to show how the options are passed to `rdrobust`.

```
. rdmc y x, c(c) pooled_opt(h(20) p(2)) verbose
Sharp RD estimates using local polynomial regression.
      Cutoff c = 0 | Left of c | Right of c | Number of obs = 2000
                  |           |           | BW type      = Manual
                  |           |           | Kernel       = Triangular
                  |           |           | VCE method   = NN
Number of obs      |       968       |      1032
Eff. Number of obs |       409       |      416
Order est. (p)     |         2       |         2
Order bias (q)     |         3       |         3
BW est. (h)        |      20.000     |      20.000
BW bias (b)        |      20.000     |      20.000
rho (h/b)          |         1.000   |         1.000
```

Outcome: y. Running variable: \_\_000002.

Method	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
Conventional	437.04	129.8	3.3671	0.001	182.643	691.441
Robust	-	-	3.0118	0.003	185.618	877.381

Cutoff-specific RD estimation with robust bias-corrected inference

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh	Weight
33	484.831	0.00	421.18	552.53	14.66	14.66	289	0.540
66	297.981	0.00	220.35	362.27	11.95	11.95	246	0.460
Weighted	398.915	0.00	348.74	445.14	.	.	535	.
Pooled	437.042	0.00	185.62	877.38	20.00	20.00	825	.

The user can also modify the options for estimation in each specific cutoff. The following syntax shows how to manually change options for the cutoff-specific estimates by setting a bandwidth of 11 in the first cutoff and 10 in the second one.

```
. gen double h = 11 in 1
(1,999 missing values generated)
. replace h = 10 in 2
(1 real change made)
. rdmc y x, c(c) h(h)
```

Cutoff-specific RD estimation with robust bias-corrected inference

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh	Weight
33	495.429	0.00	368.13	563.21	11.00	11.00	207	0.498
66	303.769	0.00	220.40	403.32	10.00	10.00	209	0.502
Weighted	399.138	0.00	321.56	455.23	.	.	416	.
Pooled	436.400	0.00	179.34	676.63	13.68	13.68	550	.

All the cutoff-specific options are passed in a similar fashion, defining a new variable of length equal to the number of cutoffs that indicates the options for each cutoff in its values. For instance, the following syntax indicates different bandwidth selection methods at each cutoff:

```
. gen bwselect = "msetwo" in 1
(1,999 missing values generated)
. replace bwselect = "certwo" in 2
(1 real change made)
. rdmc y x, c(c) bwselect(bwselect)
```

Cutoff-specific RD estimation with robust bias-corrected inference

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh	Weight
33	481.567	0.00	417.80	546.83	14.49	16.91	313	0.570
66	298.726	0.00	227.42	367.21	14.74	7.95	236	0.430
Weighted	402.969	0.00	355.30	450.28	.	.	549	.
Pooled	436.400	0.00	179.34	676.63	13.68	13.68	550	.

The `rdmc` command saves the bias-corrected estimates and variances in the matrices `e(b)` and `e(V)`, which allows for post-estimation testing using `lincom` or `test`. For instance, to test whether the effects at the two cutoffs are the same, type:

```
. rdmc y x, c(c)
```

Cutoff-specific RD estimation with robust bias-corrected inference

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh	Weight
33	484.831	0.00	421.18	552.53	14.66	14.66	289	0.540
66	297.981	0.00	220.35	362.27	11.95	11.95	246	0.460
Weighted	398.915	0.00	348.74	445.14	.	.	535	.
Pooled	436.400	0.00	179.34	676.63	13.68	13.68	550	.

```
. matlist e(b)
```

	c1	c2	weighted	pooled
y1	486.8578	291.3082	396.9415	427.9832

```
. lincom c1-c2
```

```
( 1)  c1 - c2 = 0
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
(1)	195.5496	49.3309	3.96	0.000	98.86279	292.2364

The `rdmcpplot` command jointly plots the estimated regression functions at each cutoff. The output from `rdmcpplot` is shown in Figure 1. The basic syntax is the following:

```
. rdmcpplot y x, c(c)
```

The `rdmcpplot` includes all the options available for `rdplot`. For example, the plot can be restricted to a bandwidth using the option `h()` and to use a polynomial of a specified order using the option `p()`, as shown below. This option allows the user to plot the linear fit and estimated treatment effects at each cutoff.



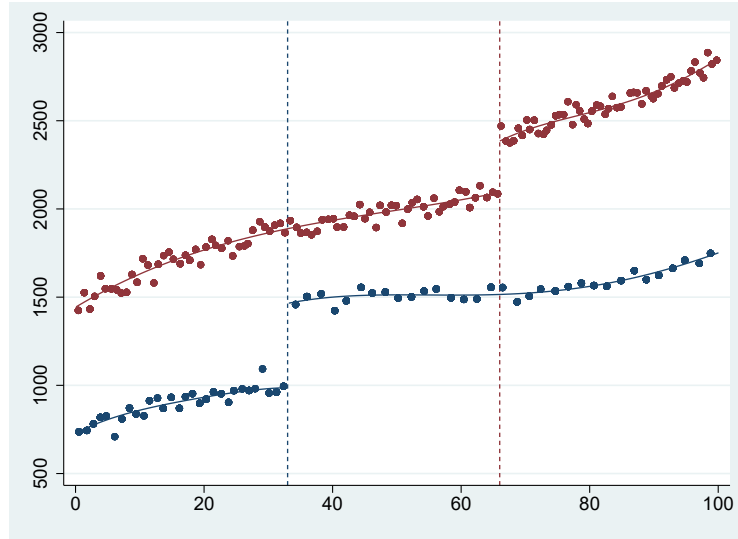


Figure 1: Multiple RD plot.

```
. gen p = 1 in 1/2
(1,998 missing values generated)
. rdmcpplot y x, c(c) h(h) p(p)
```

The resulting plot is shown in Figure 2.

The option `genvars` generates the variables required to replicate the plots by hand. This allows the user to customize the plot. The following code illustrates how to use this option to replicate Figure 2.

```
. rdmcpplot y x, c(c) genvars
. twoway (scatter rdmcpplot_mean_y_1 rdmcpplot_mean_x_1, mcolor(navy)) ///
> (line rdmcpplot_hat_y_1 rdmcpplot_mean_x_1 if t==1, sort lcolor(navy)) ///
> (line rdmcpplot_hat_y_1 rdmcpplot_mean_x_1 if t==0, sort lcolor(navy)) ///
> (scatter rdmcpplot_mean_y_2 rdmcpplot_mean_x_2, mcolor(maroon)) ///
> (line rdmcpplot_hat_y_2 rdmcpplot_mean_x_2 if t==1, sort lcolor(maroon)) ///
> (line rdmcpplot_hat_y_2 rdmcpplot_mean_x_2 if t==0, sort lcolor(maroon)), ///
> xline(33, lcolor(navy) lpattern(dash)) ///
> xline(66, lcolor(maroon) lpattern(dash)) ///
> legend(off)
```

## 6.2 Cumulative Multiple Cutoffs

We now illustrate the use of `rdms` for cumulative cutoffs using the simulated dataset `simdata_cumul`. In this dataset, the running variable ranges from 0 to 100, and units with running variable below 33 receive a certain treatment level  $d_1$  whereas units with running variable above 66 receive another treatment level  $d_2$ . In this setting, the cutoffs are indicated as a variable in the dataset, where each row indicates a cutoff.

```
. use simdata_cumul, clear
```

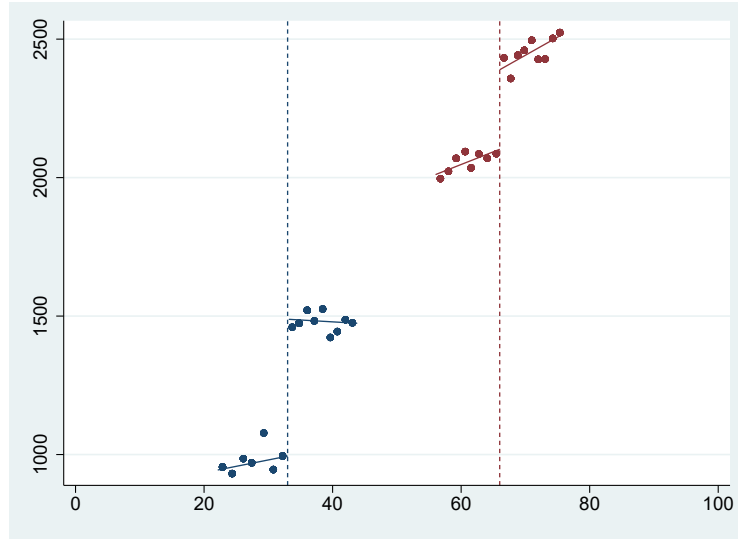


Figure 2: Multiple RD plot.

```
. sum
```

Variable	Obs	Mean	Std. Dev.	Min	Max
x	1,000	50.46639	28.69369	.0413166	99.8783
y	1,000	1508.638	488.2752	658.4198	2480.568
c	2	49.5	23.33452	33	66

```
. tab c
```

c	Freq.	Percent	Cum.
33	1	50.00	50.00
66	1	50.00	100.00
Total	2	100.00	

The syntax for cumulative cutoffs is similar to `rdmc`. The user specifies the outcome variable, the running variable and the cutoffs as follows:

```
. rdms y x, c(c)
```

Cutoff-specific RD estimation with robust bias-corrected inference

Cutoff	Coef.	P> z	[95% Conf. Int.]	h1	hr	Nh
33	395.492	0.000	363.76 423.86	15.11	15.11	286
66	342.872	0.000	315.95 373.96	12.22	12.22	265

Options like the bandwidth, polynomial order and kernel for each cutoff-specific effect can be specified by creating variables as shown below.

```
. gen double h = 11 in 1
(999 missing values generated)
. replace h = 8 in 2
```

```
(1 real change made)
. gen kernel = "uniform" in 1
(999 missing values generated)
. replace kernel = "triangular" in 2
variable kernel was str7 now str10
(1 real change made)
. rdms y x, c(c) h(h) kernel(kernel)
Cutoff-specific RD estimation with robust bias-corrected inference
```

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh
33	394.470	0.000	351.65	438.72	11.00	11.00	215
66	342.505	0.000	301.56	375.95	8.00	8.00	166

Without further information, the `rdms` command could be using any observation above the cutoff 33 to estimate the effect of the first treatment level  $d_1$ . This implies that some observations in the range  $[66, 100]$  are used. But these observations receive the second treatment level,  $d_2$ . This feature can result in inconsistent estimators for  $\tau_1$ . To avoid this problem, the user can specify the range of observations to be used around each cutoff. In this case, we can restrict the range at the first cutoff (33) to go from 0 to 65.5, to ensure that no observations above 66 are used, and the range at the second cutoff (66) to go from 33.5 to 100. This can be done as follows.

```
. gen double range_l = 0 in 1
(999 missing values generated)
. gen double range_r = 65.5 in 1
(999 missing values generated)
. replace range_l = 33.5 in 2
(1 real change made)
. replace range_r = 100 in 2
(1 real change made)
. rdms y x, c(c) range(range_l range_r)
Cutoff-specific RD estimation with robust bias-corrected inference
```

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh
33	394.698	0.000	356.12	430.45	10.96	10.96	214
66	342.180	0.000	312.20	372.04	11.18	11.18	246

The pooled estimate can be obtained using `rdmc`. For this, we need to assign each unit in the sample a value for the cutoff. One possibility is to assign each unit to the closest cutoff. For this, we generate a variable named `cutoff` that equals 33 for units with score below 49.5 (the middle point between 33 and 66), and equals 66 for units above 49.5.

```
. gen double cutoff = c[1]*(x<=49.5) + c[2]*(x>49.5)
. rdmc y x, c(cutoff)
Cutoff-specific RD estimation with robust bias-corrected inference
```

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh	Weight
33	389.528	0.00	332.94	443.69	6.26	6.26	119	0.531
66	341.015	0.00	300.39	377.33	5.04	5.04	105	0.469
Weighted	366.788	0.00	330.63	399.64	.	.	224	.

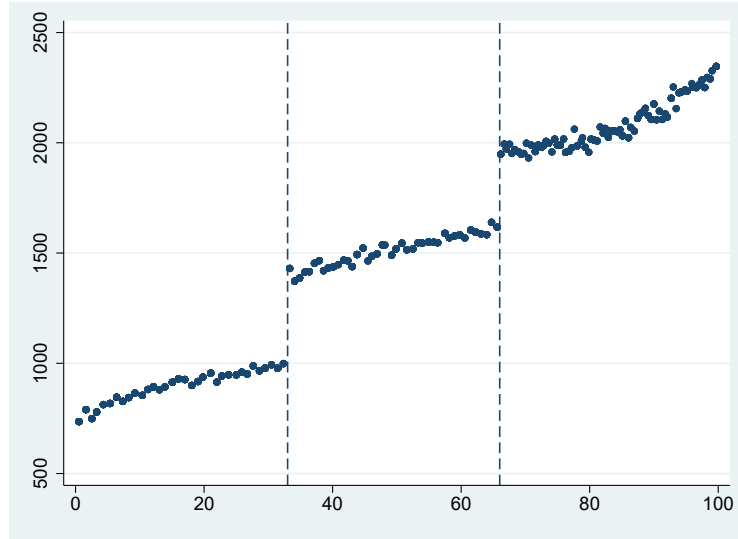


Figure 3: Cumulative cutoffs.

Pooled	363.968	0.00	180.11	551.78	8.14	8.14	333	.
--------	---------	------	--------	--------	------	------	-----	---

Finally, we can use the variable `cutoff` to plot the regression functions using `rdmcpplot`, shown in Figure 3

```
. gen binsopt = "mcolor(navy)" in 1/2
(998 missing values generated)
. gen xlineopt = "lcolor(navy) lpattern(dash)" in 1/2
(998 missing values generated)
. rdmcpplot y x, c(cutoff) binsoptvar(binsopt) xlineopt(xlineopt) nopoly
```

### 6.3 Multiple Scores

We now illustrate the use of `rdms` to analyze RD designs with two running variables using the simulated dataset `simdata_multis`. In this dataset, there are two running variables, `x1` and `x2`, ranging between 0 and 100, and units receive the treatment when  $x1 \leq 50$  and  $x2 \leq 50$ . We look at three cutoffs on the boundary: (25,50), (50,50) and (50,25).

```
. use simdata_multis, clear
. sum
```

Variable	Obs	Mean	Std. Dev.	Min	Max
x1	1,000	50.22881	28.87868	.6323666	99.94879
x2	1,000	50.63572	29.1905	.0775479	99.78458
t	1,000	.223	.4164666	0	1
y	1,000	728.5048	205.5627	329.4558	1372.777
c1	3	41.66667	14.43376	25	50

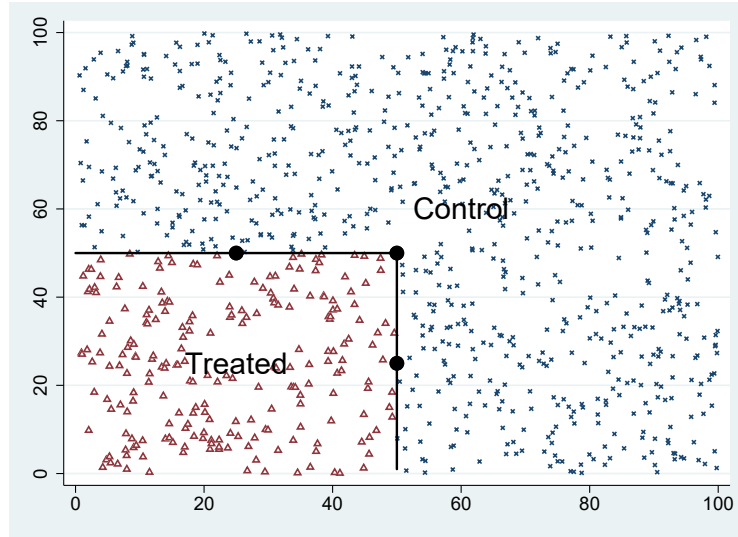


Figure 4: Bivariate score.

```

      c2 |          3    41.66667    14.43376          25          50
. list c1 c2 in 1/3

```

	c1	c2
1.	25	50
2.	50	50
3.	50	25

The following code provides a simple visualization of this setting, shown in Figure 4:

```

. gen xaux = 50 in 1/50
(950 missing values generated)
. gen yaux = _n in 1/50
(950 missing values generated)
. twoway (scatter x2 x1 if t==0, msize(small) mfcolor(white) msymbol(X)) ///
>       (scatter x2 x1 if t==1, msize(small) mfcolor(white) msymbol(T)) ///
>       (function y = 50, range(0 50) lcolor(black) lwidth(medthick)) ///
>       (line yaux xaux, lcolor(black) lwidth(medthick)) ///
>       (scatteri 50 25, msize(large) mcolor(black)) ///
>       (scatteri 50 50, msize(large) mcolor(black)) ///
>       (scatteri 25 50, msize(large) mcolor(black)), ///
>       text(25 25 "Treated", size(vlarge)) ///
>       text(60 60 "Control", size(vlarge)) ///
>       legend(off)

```

The basic syntax is the following:

```

. rdms y x1 x2 t, c(c1 c2)
Cutoff-specific RD estimation with robust bias-corrected inference

```

Cutoff	Coef.	P> z	[95% Conf. Int.]	hl	hr	Nh
--------	-------	------	------------------	----	----	----

(25,50)	243.842	0.111	-50.93	491.18	11.12	11.12	42
(50,50)	578.691	0.000	410.83	764.88	13.83	13.83	47
(50,25)	722.444	0.000	451.49	1060.15	10.83	10.83	38

Information to estimate each cutoff-specific estimate can be provided as illustrated before. For instance, to specify cutoff-specific bandwidths:

```
. gen double h = 15 in 1
(999 missing values generated)
. replace h = 13 in 2
(1 real change made)
. replace h = 17 in 3
(1 real change made)
. rdms y x1 x2 t, c(c1 c2) h(h)
Cutoff-specific RD estimation with robust bias-corrected inference
```

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh
(25,50)	336.121	0.233	-119.35	491.36	15.00	15.00	87
(50,50)	583.047	0.000	501.94	1101.24	13.00	13.00	42
(50,25)	620.692	0.000	464.92	1159.99	17.00	17.00	86

Finally, the `xnorm` option allows the user to specify the normalized running variable to calculate a pooled estimate. In this case, we define the normalized running variable as the closest perpendicular distance to the boundary defined by the treatment assignment, with positive values indicating treated units and negative values indicating control units.

```
. gen double aux1 = abs(50 - x1)
. gen double aux2 = abs(50 - x2)
. egen xnorm = rowmin(aux1 aux2)
. replace xnorm = xnorm*(2*t-1)
(777 real changes made)
. rdms y x1 x2 t, c(c1 c2) xnorm(xnorm)
Cutoff-specific RD estimation with robust bias-corrected inference
```

Cutoff	Coef.	P> z	[95% Conf. Int.]		hl	hr	Nh
(25,50)	243.842	0.111	-50.93	491.18	11.12	11.12	42
(50,50)	578.691	0.000	410.83	764.88	13.83	13.83	47
(50,25)	722.444	0.000	451.49	1060.15	10.83	10.83	38
Pooled	447.017	0.000	389.33	496.85	12.73	12.73	433

## 7 Conclusion

We introduced the **Stata** package `rdmulti` to analyze RD designs with multiple cutoffs or scores. A companion R function with the same syntax and capabilities is also provided.

## 8 Acknowledgments

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## 9 References

- Brollo, F., T. Nannicini, R. Perotti, and G. Tabellini. 2013. The Political Resource Curse. *American Economic Review* 103(5): 1759–96.
- Calonico, S., M. D. Cattaneo, and M. H. Farrell. 2018. On the Effect of Bias Estimation on Coverage Accuracy in Nonparametric Inference. *Journal of the American Statistical Association* 113(522): 767–779.
- . 2020a. Coverage Error Optimal Confidence Intervals for Local Polynomial Regression. arXiv:1808.01398 .
- . 2020b. Optimal Bandwidth Choice for Robust Bias Corrected Inference in Regression Discontinuity Designs. *Econometrics Journal*, forthcoming .
- Calonico, S., M. D. Cattaneo, M. H. Farrell, and R. Titiunik. 2017. **rdrobust**: Software for Regression Discontinuity Designs. *Stata Journal* 17(2): 372–404.
- . 2019. Regression Discontinuity Designs Using Covariates. *Review of Economics and Statistics* 101(3): 442–451.
- Calonico, S., M. D. Cattaneo, and R. Titiunik. 2014a. Robust Data-Driven Inference in the Regression-Discontinuity Design. *Stata Journal* 14(4): 909–946.
- . 2014b. Robust Nonparametric Confidence Intervals for Regression-Discontinuity Designs. *Econometrica* 82(6): 2295–2326.
- . 2015a. Optimal Data-Driven Regression Discontinuity Plots. *Journal of the American Statistical Association* 110(512): 1753–1769.
- . 2015b. **rdrobust**: An R Package for Robust Nonparametric Inference in Regression-Discontinuity Designs. *R Journal* 7(1): 38–51.
- Cattaneo, M. D., and J. C. Escanciano. 2017. *Regression Discontinuity Designs: Theory and Applications (Advances in Econometrics, volume 38)*. Emerald Group Publishing.
- Cattaneo, M. D., N. Idrobo, and R. Titiunik. 2019a. *A Practical Introduction to Regression Discontinuity Designs: Foundations*. Cambridge Elements: Quantitative and Computational Methods for Social Science, Cambridge University Press.
- . 2020. *A Practical Introduction to Regression Discontinuity Designs: Extensions*. Cambridge Elements: Quantitative and Computational Methods for Social Science, Cambridge University Press (to appear).

- Cattaneo, M. D., M. Jansson, and X. Ma. 2018. Manipulation Testing based on Density Discontinuity. *Stata Journal* 18(1): 234–261.
- Cattaneo, M. D., L. Keele, R. Titiunik, and G. Vazquez-Bare. 2016a. Interpreting Regression Discontinuity Designs with Multiple Cutoffs. *Journal of Politics* 78(4): 1229–1248.
- . 2021. Extrapolating Treatment Effects in Multi-Cutoff Regression Discontinuity Designs. *Journal of American Statistical Association*, forthcoming .
- Cattaneo, M. D., R. Titiunik, and G. Vazquez-Bare. 2016b. Inference in Regression Discontinuity Designs under Local Randomization. *Stata Journal* 16(2): 331–367.
- . 2017. Comparing Inference Approaches for RD Designs: A Reexamination of the Effect of Head Start on Child Mortality. *Journal of Policy Analysis and Management* 36(3): 643–681.
- . 2019b. The Regression Discontinuity Design. In *Handbook of Research Methods in Political Science and International Relations*, ed. L. Curini and R. J. Franzese. Sage Publications, forthcoming.
- . 2019c. Power Calculations for Regression Discontinuity Designs. *Stata Journal* 19(1): 210–245.
- Chay, K. Y., P. J. McEwan, and M. Urquiola. 2005. The Central Role of Noise in Evaluating Interventions That Use Test Scores to Rank Schools. *American Economic Review* 95(4): 1237–1258.
- Keele, L. J., and R. Titiunik. 2015. Geographic Boundaries as Regression Discontinuities. *Political Analysis* 23(1): 127–155.
- Keele, L. J., R. Titiunik, and J. Zubizarreta. 2015. Enhancing a Geographic Regression Discontinuity Design Through Matching to Estimate the Effect of Ballot Initiatives on Voter Turnout. *Journal of the Royal Statistical Society: Series A* 178(1): 223–239.
- Matsudaira, J. D. 2008. Mandatory summer school and student achievement. *Journal of Econometrics* 142(2): 829 – 850.
- Papay, J. P., J. B. Willett, and R. J. Murnane. 2011. Extending the regression-discontinuity approach to multiple assignment variables. *Journal of Econometrics* 161(2): 203–207.
- Reardon, S. F., and J. P. Robinson. 2012. Regression discontinuity designs with multiple rating-score variables. *Journal of Research on Educational Effectiveness* 5(1): 83–104.
- Wong, V. C., P. M. Steiner, and T. D. Cook. 2013. Analyzing Regression-Discontinuity Designs With Multiple Assignment Variables A Comparative Study of Four Estimation Methods. *Journal of Educational and Behavioral Statistics* 38(2): 107–141.



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